Postgraduate Certificate in Higher Education

Nottingham Trent University

# Module 1 - Developing the Reflective Academic Practitioner

## Assignment

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# PART A

# Evaluation report on own teaching

# or

# Deep learning of mathematical concepts

## Background

Scientific Mathematics is a basic course in mathematics aimed at science students who have a requirement for some areas of mathematics covered at A-Level but do not hold a qualification at that level, or who have a qualification at that level from some years ago. In reality, however, several students have a recent A-Level. An important distinction to note is that the students are studying the application of science to forensics, business and computing, rather than engaged in science or mathematics for its own sake.

Croft (2002) considers the teaching of mathematics to "non-specialists", those "students who come to university to read subjects other than mathematics ... but for whom mathematics is nevertheless a compulsory part of their university experience" (p. 147). Since mathematics is a hierarchical subject that continually builds on what has gone before, Croft states the importance of prerequisite knowledge. He suggests that teaching "students with A-level mathematics and GCSE mathematics at grade C" together "contradicts the tenet that pre-requisite knowledge should always be taken into account" (p. 145). This is a basic underpinning of the module Scientific Mathematics and the resources are not available to 'stream' the students into multiple groups. It is interesting to note, then, that attendance is poor with the frequent attendees those with either no A-Level or no recent A-Level; the course is not targeted at those with more advanced prerequisites.

Croft also points out situations where science students "are learning basic mathematical techniques when they come to university, at the same time as (and often after) they are required to apply these techniques in their main subjects" (p. 145). As we got further into the semester this became more acute, with several of the students' other modules already having finished teaching.

The session in which I was observed was week 10, entitled "Integration as anti-differentiation." In the previous 9 sessions the students have covered a variety of mathematical topics, notably including differentiation, which can be thought of as the reverse process to integration. This is the first introduction to the concept of integration. In practice, at this level, integration is the application of a simple set of rules. It may be desirable for students to have some understanding of the deeper concepts behind integration, so they get an idea of what application of the rules means. Integration is closely linked to concepts of infinity and summation of infinite series, an area in which concepts can be counter-intuitive.

There are two ways commonly used to consider integration. One is as the reverse process to differentiation, the 'other half' of Calculus. The other is to consider integration as an extension to summation; the process of calculating an infinite sum. Since the students are not mathematicians, the concept of summation would likely be as alien as integration, so not a natural vessel through which to teach a new concept. The students have spent several weeks studying differentiation, so this is the natural route for these students. However, this approach can lose the sense of what integration is actually doing, at least until it is followed up later by some 'area under a curve' work.

Wells (2007) regards that mathematics topics should be taught with an appreciation of the "background, setting, context or perspective". Kahn and Kyle (2002) regard that: "learning and teaching in mathematics and its applications clearly need to be rooted within the subject matter itself" (p. 205). The placing of topics in context is a constant tension in Scientific Mathematics, since the students are from a wide range of subject areas there is not usually one example that will appeal to all students.

Kahn and Kyle state, "simply giving lectures in which ideas are presented in a deductive order and then requiring students to solve problems is unlikely to suffice" and list "the role of historical insight" in a range of measures to engage students with the process of learning (p.207). However, Bligh (1998), writing from a general perspective, regards that introducing a topic in lecture by "giving a short history of the development of the subject" is "inappropriate for lectures," saying this "[does] not inspire students, and the significance of the history is often incomprehensible when the background issues are not known."

In the introductory lecture in differentiation, the method of explaining the deep concepts made a lot of reference to abstract concepts such as tangents and graphs. That lecture began with the derivation from first principles of a basic function. The students did not understand that this part of the lecture was illustrative and asked if it is necessary to complete this procedure for every question! I had to explain that what I was showing was a derivation and need not be completed every time, and I think the deep concepts were lost in the confusion.

It may be worth noting that integration is generally well known as a difficult topic, and having looked ahead in the notes the students had expressed apprehension at this.

## Planning the session

With the need for context and deep understanding in mind, I was looking for a way to introduce the concepts behind integration – those of infinity and infinite sums – without getting distracted in worried note-taking on the part of the students. Over Easter, I attended a Sixth Form Outreach event which included a talk on concepts of infinity (Broomhead, 2008). This used Xeno's paradox of Achilles and the Tortoise to introduce the concept of an infinite sum, and Broomhead used the analogy of a bouncing ball to illustrate the concept, although he did not have such a ball with which to demonstrate. Broomhead said a lack of resolution of this counter-intuitive paradox is credited with setting back the development of the Calculus until after the Renaissance. If the students can gain some appreciation of the paradox they may be better equipped to deal with the deep concepts behind integration. Using this historical and philosophical device may be a useful introduction to this lecture, especially given the lack of a clear application appropriate for all students, but note this would be taking the recommendations of Wells (2007) and Kahn and Kyle (2002) over those of Bligh (1998).

I decided this idea was an appropriate one in my circumstances. It would allow an explanation at the start of the lecture with the students able to consider some of the deep concepts behind infinite sums in a setting which they are obviously not going to be expected to reproduce in exam conditions. This would lead into an outline of the concept of integration and then an example. Involving the movements of a tortoise and a bouncing ball, this might even be fun and break the anxiety over this difficult topic.

In advance of the lecture I sketched an outline for this demonstration. I would detail the paradox and its implications, revisit differentiation and show how this familiar process can be reversed (through deduction and trial and error) to produce integration. It is then necessary to define some notation and basic rules and then the students should be ready to have a go at some problems. The lecture was expected to take 10-20 minutes to be followed by a problem class section which would take approximately 1 hour, determined by student need.

In the problem class section of the lesson the students work at their own pace through a set of problems. This is a course in applications of mathematics and as such is a practical undertaking. It is necessary for the students to undertake sufficient practice at the techniques learned to encourage confidence and memory. Bligh (1998) regards lectures alone as not effective in teaching practical skills, beyond the "information component" that is a necessary preamble to most skills learning. In this case, and mathematics is certainly such a case, Bligh expects "a presentation, such as a lecture, before behavioural practice to be an effective combination." d'Inverno (2003) believes it is important for the students to do some writing of mathematics, since "it is important that students develop a 'tactile' familiarity with mathematics simply by shuffling the symbols around," although he regards as weak the process of lecturing whereby the lecturer writes problems onto the board and the students copy them down. Wakeley (2004) believes mathematics must be practiced to be learned. A short introduction followed by a problems session seems to be indicated as an effective method.

As the students are leading the pace of this session, they usually build in a natural off-topic discussion which serves to break up the problem class. d'Inverno believes "anything which breaks up a lecture is ... good news," giving students "a chance to take a breather and to refocus." Unfortunately there is insufficient material to break into multiple lecturer demonstrations, as would be preferred to provide variety in the session.

## The session

Scientific Mathematics takes place in a flat lecture theatre with fixed chairs at long benches. The peer observer regarded the room as "not ideal" and this is certainly the case. Such a layout does not facilitate student conversation and provides practical difficulties when walking between students, particularly when a number of students are present.

Only 2 students were in attendance, in keeping with recent weeks but a surprise given several students expressions of anxiety over integration. One is a mature student returning after a period in employment to complete a second degree who has covered the material before in that degree but does not remember it well. Call her A. The other is straight from school and has not covered the material before, having only GCSE mathematics. Call her B. They are both highly capable but lack confidence. Both students scored very well in the recent coursework submission. The observer commented that having only 2 students "meant [Peter] could give them plenty of individual attention."

### *Lecture section*

The lecture section of the session broadly followed the notes I had made in content, with pace and an occasional aside determined by interaction with the students. The observer noted "Peter asked for student feedback" during this section.

I mentioned the previous work on differentiation to cause a recall and provide a link to previous material. The observer noted that I linked this material to previous work, allowing "good connections between topics," but felt a more detailed and explicit set of learning outcomes might be more useful to give the learners an idea of the path the session was going to take and "provide a route through the session for Peter." However, Cox (2004) reports that it is unusual to begin a mathematics lecture with a detailed lesson plan, rather a general understanding of the form the session will take. It is difficult to give too detailed a description on such a small amount of content, but I could have told them we were going to try a bit of mathematical reasoning followed by an example on integration and then a problem class. The observer also noted, "I think this type of workshop style teaching might not always lend itself to a linear process of intro, development and conclusion because of the nature of individualised work" and reported despite the lack of a formal plan "the learning aims and outcomes ... were met in the session."

Xeno's paradox produced a bemused but thoughtful response from the students and the reasoning behind reversing differentiation was well met. The observer reported "Peter talked with knowledge ... and was clearly enthusiastic about it."

The students certainly engaged with the richer Xeno's paradox metaphor much more than they had with the more abstract introduction to differentiation in terms of tangents to curves. The observer noted that this section was engaging and generated questions from students, believing the "Q and A technique" to be an effective one.

### *Problem class section*

Student A got on with the exercises fairly confidently. She asked a couple of questions and was capable. She noticed an error in the exercise solutions.

Student B was much less confident. She said she didn't understand the material several times even though her behaviour suggested she did but she just didn't have the confidence in what she was doing. The observer noted, "[Peter] worked well with her encouraging her ... to come to an understanding herself." After a few exercises she grew more in confidence.

The students sat much further apart than usual, which meant I could follow one student or the other, but not both. I moved between the two and this seemed to work fairly well. The observer noted, "[Peter] was consistent in his approach to both students," that, "Peter knew both his students well and what their particular needs, abilities and experiences were," and that, "he was able to reassure and support both students in ways that were meaningful to them."

On the use of the problem class method, the observer noted that the "workshop/workbook" method with the students working at their own pace was appropriate, noting this "allowed students less confident in their abilities to work to their needs without being the focus of the whole group." The observer reported that in response to student queries, "[Peter] encouraged questioning ... and supported students working things out themselves through this questioning rather than giving them the answers," and that even though the students were working through a workbook, "there were discussions between all."

Student A finished the exercises much earlier than student B. I said she could read through next week's material or go early. I didn't want to start the next week's material on the board because student B was not ready, though I did not say so of course. Student A read through next week's material, said it was familiar and she would wait for next week, and left early. Some further, optional problems might have been useful here, as I got the sense full confidence had not been attained. The observer wondered if additional resources via websites and materials on the VLE would "extend and develop further independent learning outside of the session."

Student B continued through the exercises and by the end was completing them at an acceptable pace. She lacks confidence. I told her she did understand the material but just hadn't practised enough yet, as she was with differentiation. Again, some optional take-home material might have helped here.

At a point suitable to themselves, as usual, the students wandered into a non-mathematical discussion. The observer felt a formal break might have been more appropriate. The observer noted that small groups can be "quite difficult to manage" but "the teaching strategies and resources worked well in this case allowing for good working relationships."

## Suggestions for future development

The observer suggested: that I provide a formal session outline and "clearer signposting between elements of the session"; that through my enthusiasm I may have been speaking too quickly, though noting "your natural enthusiasm ... is a very effective way of engaging your students"; I should face the class and not the whiteboard when explaining; possible inclusion of group work; and, inclusion of formal time for a break.

## Deep learning vs. a black box

With the content of this session, and the similar one on differentiation, I wondered about the importance of teaching the concepts behind the mathematics. As a mathematician, I have a poor appreciation of how one might learn (certainly, remember) a technique without first understanding it. Furthermore, there tends to be distaste for taking the validity of a technique at face value without seeing a derivation or mathematical proof firsthand.

However, the students I am teaching are using mathematics as a tool, a means to an end. In their home disciplines they will have problems which need to be solved using mathematical techniques and they are looking to me to teach them how to apply these techniques. Surely then teaching the procedure of applying the technique is sufficient?

Raine (2005) uses the analogy of car drivers who do not understand how the car engine works to explain this viewpoint. He regards the way mathematics has been taught to science students as procedural,

> "we got into the mode of teaching mathematical skills to science students in the way that skills are generally taught: by constant repetition and coverage of all possible variants of a given problem, eventually to the extent that mathematics becomes indistinguishable from pattern matching. And the sad thing is that you know it works – at least until you change the context and start asking for applications of the techniques in unfamiliar surroundings" (p. 14)

Fuson, Kalchman and Bransford (2005) suggest this procedural way of teaching mathematics "often overrides students' reasoning processes, replacing them with a set of rules and procedures" (pp. 217-218). They argue focusing the instruction on this procedural knowledge is ineffective and causes a disconnect from the meaning behind the mathematics, evidenced by students failing to correct erroneous answers which are clearly unrealistic.

However, Elton (1971) reports tensions in this area between mathematicians and departments whose students they teach:

> "It is often claimed by mathematicians that all the non-mathematicians want is 'tricks for the trade.' The non-mathematicians retort that this is not true, but that mathematicians will introduce mathematical rigour for its own sake, because 'it is good for you.' After that the battle develops along predictable lines." (p. 79)

Raine concludes that science students would benefit from a chance to acquire "some mathematics instead of just mathematical techniques," going so far as to suggest students are taught to use symbolic algebra software packages to conduct the mathematical techniques and concentrate the learning entirely on the deep understanding of the mathematics.

Stevens (2003), discusses this point in relation to engineers and calculus, since "most of the relevant knowledge is embedded within the major computer programs used these days" and "engineers can be confident in the output from these codes." He concludes with the opinion that knowledge of calculus is important since engineers cannot have a "blind belief" in the output of mathematical models produced by software packages. He argues that it is "imperative" they understand the mathematics underpinning the model.

Entwistle and Ramsden (1983) explored the ways students approach studying. They contrast two main types of learning: "deep" (or "meaning") and "surface" (or "reproducing") (p. 193). The deep approach being "internal," focused on the content of the article or problem and the knowledge, experience and interests of the learner. The surface approach is "external", focused on the task and its requirements. They suggest that the surface approach,

> "implies a process of learning in which alien material is to be impressed on the memory for a

> limited period and with the specific intention of satisfying external demands. There is no expectation that the content will become a continuing part of the learner's cognitive structure." (p. 195)

They notice that students relying on surface learning are interested in the courses mainly for the qualification and vocational aspects, while those using deep learning are interested in the work itself (pp. 195-6). It seems that the students of Scientific Mathematics are enrolled on vocational courses – business and computing for science and forensic sciences – and so might be expected to have a natural inclination towards surface learning. In such an analysis of student learning processes, Entwistle and Ramsden caution that most students will be using both styles at different times.

There is an expectation that a student of mathematics must not just be able to apply a technique but also understand when to apply that technique. Without deep understanding of a mathematical technique it can be difficult to apply this to new problems. Dreyfus (2002) reports the results of a study which demonstrates that such students are able to demonstrate well that they can reproduce the desired technique but that they are not able to adapt this knowledge to unfamiliar circumstances. There is also some suggestion that deep learning can help in the retention of the learning and confidence in the subject (Entwistle and Ramsden, 1983).

There seems to be a consensus, then, that not only is it important for the students to engage in deep learning but in some quarters it is regarded as *all that is important*; with the routine mathematical analysis left to computers. Fuson, Kalchman and Bransford (2005) even attribute some students' dislike of mathematics as a subject to focusing of the instruction merely on procedural knowledge.

How to achieve this deep learning seems to be a cause of differing opinions, as seen with Wells (2007) and Kahn and Kyle (2002) favouring and Bligh (1998) rejecting historical context in lectures. Burn (2002) and Dreyfus (2002) discuss the difference between the way mathematical ideas are formed and the way those ideas are later formalised and published, with Burn suggesting the way mathematics is formally taught is at odds with the psychological experiences of the student's personal journey. Burn suggests mathematical instruction based more closely on the genesis of mathematical ideas to provide the basis for developing "active learners of mathematics" (p. 21). Dreyfus particularly regards that the method of simply presenting "theorem-proof-application" "does not work for the vast majority of students ... taking mathematics as a required service subject" (pp. 26-27).

The students certainly engaged with the Xeno's paradox metaphor, but the bulk of the session focused on the procedural learning and repetition-until-confident of integration techniques. My reading around this subject leads me to concern that I have simply trained my students in the mathematical techniques of Scientific Mathematics – according to Dreyfus, "simply training them to acquire the skills of a computer algebra package" – and they will struggle to retain this knowledge beyond the exam or apply it to unfamiliar problems. Since they have received instruction in procedural techniques and not been trained to think as mathematicians do, it is possible they will have the same level of difficulty when they have to apply or relearn their mathematics that they had in learning it in the first place.

## Micro-teaching

In my micro-teaching session to my learning set I expanded on the theme of deep mathematical learning using concepts of infinity. I attempted to further increase the appeal of such a session by using a lecturing method that involved annotating electronic slides using an interactive whiteboard.

In the discussion that followed I attempted to gain views from my learning set on whether deep understanding is necessary to learning mathematics. These are non-mathematicians. The view was generally held that memory and understanding are helped greatly by deep learning of the concepts behind a technique.

# PART B

# Evaluation report on observation of teaching

# or

# Live mathematics, by chalk and talk

## Background

Mathematical Techniques is a year 1 undergraduate module taught to Physics students. My observation of the session on line integrals and scalar potentials took place towards the end of this year long module, late in semester 2. The experienced lecturer I observed noted "students are using mathematics in their physics course so the material is taught for its applicability rather than for its own sake." It is interesting to note I felt it necessary to bring this distinction to the attention of my observer also.

In difference to Scientific Mathematics, the material here was much more advanced, there was more content to cover, more students were in attendance and the session lasted only one hour.

## Planning the session

The session was planned to include periods of explanation, guided example and the students doing examples, on the basis that "mathematics must be practiced to be understood" and to allow for student participation and feedback. The lecturer gave me a lesson plan based on 5 and 10 minute intervals, although I got the sense he had not planned this formally until asked to by the pre-observation form. This included 2 iterations of the explain-example-practice cycle. Bligh (1998) studies what lectures can achieve, and in terms of skills based lessons, states, "verbal presentations present words; and words are what students get from them. If you want them to be able to do something, put them in a situation where they practice doing it" (p. 22). Wakeley (2004) notes methods used in mathematics are different from many other disciplines and observes,

> "One of my lecturers once wrote in the introductory notes to his course that 'The only way to learn maths is to do maths'. It is this rather than any formulae or trick that has stuck with me throughout my university career" (p. 8).

However, Bligh reflects that most "skills have an information component... and lectures are as effective as other methods for teaching that information." In this case, Bligh expects "a presentation, such as a lecture, before behavioural practice to be an effective combination" (pp. 22-23).

The pre-observation form asks how the extent of the student's learning will be assessed. The lecturer has answered that "if the students are answering the questions correctly they have learned to apply the technique." This suggests the students are not expected to achieve (or demonstrate) deep learning (see Entwistle and Ramsden, 1983).

## The session

At the start of the session students were given a handout. This had a printed explanation of the notes and some exercises. The availability of these printed notes meant that the lecturer did not need to spend lots of time writing on the board and the students copying them down. The important points could be highlighted on the board and the printed notes could be relied on to provide accurate detail for the students. The lecturer put the material clearly in context, making reference to previous work although the aims, objectives and outcomes of the session were not made clear.

The format of the session was that the lecturer gave some explanation of the topic in terms of the previous material and then went through an example on the board. The students were then expected to work through the relevant exercises. There were two iterations of this procedure during the session which produced a variety of activity, meaning the students remained engaged for longer.

The lecturer asked questions of the students throughout the examples on the board and felt the student interaction went well, in that the students were answering the questions. d'Inverno discusses interaction between lecturer and students during lectures, believing this breaks the lecture which gives students “a chance to take a breather and to refocus,” gives the opportunity to reinforce important points and gives an opportunity to monitor student understanding. He believes lectures should do more than simply provide a full set of notes and that student interaction can provide some of this “value added component” (p. 18-19).

I was impressed with the style of this, in that the students were carefully asked questions on the easier parts of the material, particularly that covered in previous weeks, then guided more carefully through the more difficult extension of this into new territory. This meant the students engaged with the learning and reinforced previous knowledge but made sure they did not get put off by getting questions wrong or not being able to answer. It was also well executed; if there were no student answers forthcoming, the lecturer took this into his stride and either provided a gentle encouragement, sometimes breaking the question into parts, or explained the answer.

In the post-observation meeting, the lecturer felt that some real, practical examples would have improved the session. This may be the case, especially since all the students were engaged on the same degree course. Kahn and Kyle (2002) regard that: "learning and teaching in mathematics and its applications clearly need to be rooted within the subject matter itself" (p. 205). Wells (2007) regards that topics should be taught with an appreciation of the "background, setting, context or perspective". He states,

> "Mathematics is an extraordinarily rich and imaginative activity, a science and an art rolled into one, but this richness is replaced for pupils and students at various levels of education by a dull and unimaginative dryness" (p.12).

The exercise section was useful, for the reason discussed previously that mathematics is a skill that must be practiced but also in response to a concern raised by the lecturer in the pre-observation meeting: "Were the concepts understandable?" In the example session the "Q and A" interaction between the lecturer and students was a useful device to encourage interaction and ensure the students were following the concepts, but it was noticeable that not all students answered questions. Particularly, I noticed that when the lecturer asked "Do you understand?" although there was a general affirmative murmur, not all students participated in this. However, in the exercises section of the session all students were required to attempt the exercises and this ensured that those who had not followed the explanation were brought to light. The lecturer was available during this session and assisted with some queries, although many were dealt with by informal group discussion.

## Differences and lessons from Scientific Mathematics

This session included a good amount of content and it was clear that this must be progressed at a rate to allow everything to be covered in the session. In Scientific Mathematics, sessions are longer and there is never any doubt that I will have time to explain everything in the time available, which provides a more relaxed but less focused environment. Careful planning was needed to ensure all the material could be covered in time.

The class size was much larger than in Scientific Mathematics so class management issues were different. It is very unusual for my students to be talking as I am talking, since to do so would be very obvious, yet in this session it was common for some students to be talking as the lecturer was. In Scientific Mathematics I have time to ask each student individually whether they are getting the correct answers for exercises and understanding the topic. Here the larger class size and shorter time period meant that the lecturer interaction with the exercises portion of the session was much more guided towards the students who seek help with their problems, and much informal group discussion took place to solve mutual problems. In Scientific Mathematics there are generally not enough students to have anything other than a whole class discussion, and the inclusion of groups of students working through solutions together is certainly an interesting one.

## Chalk and talk

In the post-observation session the lecturer asked "Would the use of OHP slides or PowerPoint improve the session?" 'Chalk and talk' is a term used to describe traditional teaching methods where the lecturer writes on the board live and talks through the material (although the use of a whiteboard means that no chalk is involved). Teaching by chalk and talk is seen as old fashioned in many subjects but survives in mathematics. There are two important variants of this procedure: the lecturer writes all the course content on the board and the student copies it down; or, printed notes are available and the writing on the board is used to highlight the key points. This latter case is the one used in this session and contrasts with the alternative (non-chalk and talk) method of using OHP or PowerPoint slides to highlight key points and demonstrate examples. In a sense, then, the method used here was the middle of three: completely live; part live and part pre-prepared; and, wholly pre-prepared. These options will be discussed here.

d'Inverno (2003) discusses the weaknesses of teaching techniques in which the lecturer simply writes notes on the board and students copy them down, our "completely live" scenario. This supports the methods of pre-prepared printed notes. However, d'Inverno believes it is important for the students to do some writing of mathematics, since “it is important that students develop a 'tactile' familiarity with mathematics simply by shuffling the symbols around.” d'Inverno uses incomplete printed notes to allow students to “shuffle” the mathematical symbols while “getting rid of the redundant process of copying down text.” In this session the students were given complete notes but were able to practice writing mathematics during the exercises section.

Burn (2002) discusses the difference between the way mathematical ideas are formed and the way those ideas are later formalised and published, suggesting the way mathematics is formally taught is at odds with the psychological experiences of the students personal journey. Dreyfus (2002) also discusses the differences between mathematics as it is created and mathematics as it is taught, "so often" following "theorem-proof-application" (p. 26). Dreyfus cautions a serious disadvantage in this approach: inflexibility in adapting to the students. Regarding the rote learning of definitions, and the adaption of model solutions, Kahn and Kyle report, "there are a whole range of ways in which students can avoid genuine engagement with mathematical content" (p.207).

Burn suggests mathematical instruction based more closely on the genesis of mathematical ideas to provide the basis for developing "active learners of mathematics" (p. 21). Burn suggests that students need to engage in the process of discovering mathematics. Dreyfus suggests that otherwise the students will lack,

> "the working methodology of the mathematician, that is they lack the know-how that allows them to use their knowledge in a flexible manner to solve problems of a type unknown to them ... they have been taught the products of the activity of scores of mathematicians in their final form, but they have not gained insight into the processes that have led mathematicians to create these products."

In this session, students were guided through the material by extension of previous material into new areas. This seems to allow the students the sense of discovery of new mathematics although it is possible this is not close enough to the way in which mathematics is truly discovered. However, taking the time for the students through true mathematical discovery of the material would certainly mean less material is covered in each session. Burn recognises students "cannot usually be expected to recreate the genetic process without support" (p.33). Dreyfus notes the "theorem-proof-application" method, with all its disadvantages, can be advantageous in that the students move through the syllabus at a sufficient pace to ensure all the material is covered. Burn asserts that lecturers need to find ways to expose the development of the mathematics, "rather than simply present them with deductive or formal products." It seems that the process of student answers to Q & A guiding the development of mathematical examples may be a suitable compromise between these tensions.

Wakeley (2004), writing from a student perspective, reports a view that "chalk and talk" lectures are "rather out-dated," and notices some of the pressures within universities on this method. With a poetic flair, Wakeley writes in favour of a live demonstration of mathematics, involving,

> "a living breathing mathematician, explaining the problem, using the piece of chalk as an extension of their mind, to bring the solution to life. To see mathematics ... being done live in front of you will often bring insight to the mass of symbols which appear on a printed page. The lecturer somehow provides a light through the printed algebra, to the deeper method and to the understanding which underpins mathematical thinking."

The enthusiasm here clearly shines brightly, but the point is clear that simply studying pre-prepared printed mathematics, our "wholly pre-prepared" scenario, is not sufficient, Wakeley feels that seeing the mathematics performed live is a vital component of learning mathematics.

This brings me to an interesting incident. During one of the examples in this session the lecturer made a mistake. This might be taken as a sign that pre-prepared lecture slides would be more appropriate, but I will argue differently. The mistake was made, realised when the next calculation did not seem 'right' and the lecturer went back and corrected the mistake. I think that this is a good insight into the process of true mathematics and would fit well with the views of Burn (2002) and Dreyfus (2002). I can imagine the students getting a sense that the lecturer is 'only human' and of the mathematics as 'something *I* could do too.' I would contrast this with pre-prepared, perfected slides which are displayed, in which the process of conducting the mathematical technique is cold and clinical.

Baxter (2005), as a new lecturer, tried teaching from pre-prepared slides, but reports, "with ready prepared slides, it is all too easy to rush through material, leaving the audience confused. Writing notes on the board gives a natural rhythm to the lecture that prepared slides do not." Baxter also found that while he thought adding detail and explanation would aide understanding, he found that "too much detail can be distracting." Giving the extra detail in pre-prepared handout notes might be advantageous here.

Wakeley (2004) also attributes value to the "talk" portion of "chalk and talk," giving importance to what the lecturer says as he is performing the mathematics. Wakeley concludes mathematics is "best taught by someone working at a chalkboard, ready for battle – armed with a sharp mind and a stick of chalk".

It seems then that the middle scenario, lectures that are "part live and part pre-prepared," the option taken in this session, is the most appropriate to the teaching of mathematics.

## *Accessibility*

Chalk and talk, the live demonstration of mathematics, is by its very nature going to be difficult to access for students with disabilities who cannot immediately access the written or oral content. Having printed handouts, made available in a suitable format in advance of the lecture to those students who need them, will go some way to improving the situation. But there will always be the problem that students with visual impairments or dyslexia may not be able to access the 'chalk' and students with hearing impairments may not be able to access the 'talk'.

## *Interactive whiteboards*

Starkings and Krause (2007) report on a "new type" of "chalk and talk," the interactive whiteboard. This replaces the projection screen and provides a touch screen interface to a computer. Any content which can be displayed through a projector can be displayed on the interactive whiteboard, with the added advantage that physically touching the screen replaces mouse actions and 'pens' can be used to 'write on' the board (really simulated through software), meaning this board can be used as a traditional whiteboard, a touch screen interface to a computer or, crucially, a combination of the two.

Starkings and Krause report a favourable initial reaction to this method from students.

Our scenario of lectures that are chalk and talk, part live and part pre-prepared, regarded as a powerful mechanism for teaching mathematics, can be brought to a whole new level by the use of an interactive whiteboard, allowing pre-prepared (possibly incomplete) lecture slides to be annotated as one would normally write on a board. Additionally, the software can store on screen writing for later distribution, circumventing some of the accessibility issues[1] and allowing access to the 'chalk and talk' content for students who wish to revise the lecture at a later date.

## Micro-teaching

I used an interactive whiteboard in my micro-teaching exercise to my learning set. I prepared a series of PowerPoint slides around concepts of infinity but deliberately left these incomplete. When dealing with Xeno's paradox, for instance, the slides contained an explanation of the premise and a drawing of the race course, but I left drawing and moving around Achilles and the tortoise until the live session. Similarly, when following a diagrammatical approach to the summation of an infinite series I left the detail to be completed live. I think this is a powerful method, since it gives some structure and clarity to the presentation while still allowing for the live demonstration of mathematics, and it allows the live written content to be stored for later distribution.

This method got an enthusiastic response from my learning set with most feeling it was appropriate although one person felt the technology was distracting and confusing. On the subject of live demonstrations of mathematics, most students agreed seeing the material performed live was an advantage, although one felt that seeing a lecturer "rattle off" all the mathematics without careful pre-preparation would give her a sense of hopelessness that she would never be able to do the same. My learning set are all non-mathematicians.

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[1] The image of the stored writing can be enlarged, but mathematics is difficult to automatically process into text appropriate for conversion to speech.

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